GRAVITATIONAL AND COSMOLOGICAL PROPERTIES OF A BRANE-UNIVERSE

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The aim of this contribution is to provide a short introduction to recently investigated models in which our accessible universe is a four-dimensional submanifold, or brane, embedded in a higher dimensional spacetime and ordinary matter is trapped in the brane. I focus here on the gravitational and cosmological aspects of such models with a single extra-dimension.

The traditional view on extra-dimensions is the Kaluza-Klein picture: the matter fields live in compact (usually flat) extra-dimensions, and their Fourier expansion along the extra-coordinates lead to an infinite collection of so-called Kaluza-Klein modes, which can be interpreted as four-dimensional fields. Their mass spectrum is very specific since it is discrete with a mass gap of the order of R^{-1}, where R is the size of the extra dimensions (common for all extra-directions in the simplest cases). The non-observation of Kaluza-Klein modes in present collider experiments therefore provides an upper bound on the size of the extra-dimensions:

$$R \lesssim 1 \ (\text{TeV})^{-1}. \tag{1}$$

Superstring theory requires extra-dimensions to be consistent at the quantum level and the Kaluza-Klein compactification was invoked to get rid of the superfluous six extra-dimensions, until a new picture on extra-dimensions emerged recently, such as in the Horava-Witten eleven-dimensional supergravity[1]. In this context, the ordinary matter fields are not supposed to be defined everywhere but, in contrast, are assumed to be *confined* in a submanifold, called *brane*, embedded in a higher dimensional space.

The Horava-Witten model was followed by the radical proposal of Arkani-Hamed, Dimopoulos and Dvali[2], who, in order to solve the hierarchy problem, suggested that we live confined in a three-brane surrounded by $n \geq 2$ (flat and compact) extra-dimensions with a size R as large as the millimeter scale. One could then explain the huge value of the Planck mass, with respect to the TeV scale, simply as *a projection effect*, the relation between the fundamental (higher-dimensional) Planck mass $M_{(4+n)}$ and the usual four-dimensional Planck mass $M_{(4)}$ being given

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by

$$M_{(4)}^2 \sim M_{(4+n)}^{2+n} R^n. \tag{2}$$

The absence of any observed deviation from ordinary Newton's law gives an upper bound on the compactification radius [3], presently of the order of a fraction of millimiter ($R \lesssim 0.2\,\mathrm{mm}$).

Another proposal, even more interesting from the point of view of general relativity and cosmology, is due to Randall and Sundrum [4]. They consider only one extra-dimension but take into account the self-gravity of the brane endowed with a tension σ. They moreover assume the presence of a negative cosmological constant Λ in the bulk (thus Anti-de Sitter). Provided the tension of the brane is adjusted so that

$$\frac{\kappa^2}{6}\sigma = \frac{1}{\ell} \equiv \sqrt{-\Lambda/6}, \tag{3}$$

they find a static solution (and mirror-symmetric with respect to the brane) of the five-dimensional Einstein equations, described by the metric

$$ds^2 = e^{-2|y|/\ell}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{4}$$

where $\eta_{\mu\nu}$ is the usual Minkowski metric. Linearized gravity in the brane can be worked out explicitly in this model [4,5] and one finds that the effective gravitational potential reads

$$V(r) = \frac{G_{(4)}}{r}\left(1 + \frac{2\ell^2}{3r^2}\right), \tag{5}$$

where the four-dimensional gravitational coupling is given by

$$8\pi G_{(4)} = \kappa^2/\ell. \tag{6}$$

The (approximate) recovery of the usual Newton's law is due to the presence of a zero mode which is (exponentionally) localized near the brane. The corrections to the usual potential, significant only at the AdS scale ℓ and below, arise from a continuum of massive graviton modes.

Let us now turn to cosmology. The main motivation for exploring cosmology in models with extra-dimensions is that the new effects might be significant only at very high energies, i.e. in the very early universe. Let us thus consider a five-dimensional spacetime with three-dimensional isotropy and homogeneity, which contains a three-brane corresponding to our universe. The metric can be written in the form

$$ds^2 = -n(t,y)^2 dt^2 + a(t,y)^2 \delta_{ij}dx^i dx^j + dy^2, \tag{7}$$

with the brane (here spatially flat) located at $y = 0$.

The energy-momentum tensor can be decomposed into a bulk energy-momentum tensor, which is assumed to vanish here, and a brane energy-momentum tensor, the latter being of the form

$$T_B^A = S_B^A \delta(y) = \{-\rho_b, p_b, p_b, p_b, 0\}\delta(y), \tag{8}$$

where the delta function expresses the confinement of matter in the brane and where ρ_b and P_b are respectively the total energy density and pressure in the brane and depend only on time. The presence of the brane induces a jump of the extrinsic curvature tensor (defined by $K_{AB} = h_A^C \nabla_C n_B$, where n^A is the unit vector normal to the brane) related to the brane matter content according to the Israel junction conditions

$$\left[K_B^A - K\delta_B^A\right] = \kappa^2 S_B^A. \tag{9}$$

Still assuming, for simplicity, mirror (i.e. Z_2) symmetry, these junction conditions applied to the cosmological metric (7) yield the two conditions

$$\left(\frac{n'}{n}\right)_{0+} = \frac{\kappa^2}{6}\left(3p_b + 2\rho_b\right), \qquad \left(\frac{a'}{a}\right)_{0+} = -\frac{\kappa^2}{6}\rho_b. \tag{10}$$

The bulk Einstein equations can be integrated [6], and the substitution of (10) at the location of the brane gives the *generalized Friedmann equation*

$$H_0^2 \equiv \frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^4}{36}\rho_b^2 + \frac{\Lambda}{6} + \frac{\mathcal{C}}{a^4}, \tag{11}$$

where $\mathcal{C}$ is an integration constant, the subscript '0' denoting evaluation at $y = 0$. The most remarkable feature of (11) is that the energy density of the brane enters quadratically on the right hand side in contrast with the standard four-dimensional Friedmann equation where the energy density enters linearly. As for the energy conservation equation it is unchanged in this five-dimensional setup and still reads

$$\dot{\rho}_b + 3H(\rho_b + p_b) = 0. \tag{12}$$

In the simplest case where $\Lambda = 0$ and $\mathcal{C} = 0$, the evolution of the scale factor is given[7] by $a \sim t^{1/4}$ (instead of $a \sim t^{1/2}$) for radiation and $a \sim t^{1/3}$ (instead of $a \sim t^{2/3}$) for pressureless matter. Such behaviour is problematic because it cannot be reconciled with the primordial nucleosynthesis scenario, wich relies on the competition between microphysical reaction rates and the expansion rate of the universe.

Brane cosmology can however be made consistent with nucleosynthesis [8] if, following Randall and Sundrum, one introduces a negative cosmological constant in the bulk as well as a tension in the brane so that the total energy density in the brane, ρ_b, splits into

$$\rho_b = \sigma + \rho, \tag{13}$$

where ρ is the usual cosmological energy density. Substituting this decomposition into (11), one gets

$$H^2 = \left(\frac{\kappa^4}{36}\sigma^2 - \frac{1}{\ell^2}\right) + \frac{\kappa^4}{18}\sigma\rho + \frac{\kappa^4}{36}\rho^2 + \frac{\mathcal{C}}{a^4}. \tag{14}$$

If one fine-tunes the brane tension and the bulk cosmological cosmological constant as in (3), the first term on the right hand side vanishes. The second term then

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becomes the dominant term if ρ is small enough and *one thus recovers the usual Friedmann equation at low energy*, with the identification $8\pi G = \kappa^4 \sigma/6$, which is consistent with (3) and (6).

The third term on the right hand side, quadratic in the energy density, provides a *high-energy correction* to the Friedmann equation which becomes significant when the value of the energy density approaches the value of the tension σ and dominates at higher energy densities. Finally, the radiation-like term, proportional to $\mathcal{C}$, is related to the bulk Weyl tensor. It must be small enough during nucleosynthesis in order to satisfy the constraints on the number of extra light degrees of freedom.

Our model is valid if nucleosynthesis takes place in the low energy regime, i.e. $\sigma^{1/4} \gtrsim 1$ MeV, which implies $M \equiv \kappa^{-2/3} \gtrsim 10^4$ GeV for the fundamental mass. However, the requirement to recover ordinary gravity down to scales of the submillimeter order gives the tighter constraint

$$M \gtrsim 10^8 \text{ GeV}. \tag{15}$$

With this viable *homogeneous* cosmological model, the next step is to investigate the perturbations from homogeneity, to check whether brane cosmology can be made compatible with the current observations and more interestingly, whether it can provide deviations from the standard predictions which might be tested in the future. Both questions are still unanswered today. One can however get a first flavour of the possible modifications by analyzing the equations obtained from the linearized five-dimensional equations [9]. One finds equations similar to the standard evolution equations for the perturbations supplemented with corrective terms, which are of two types: new terms due to the modified Friedmann equation, which become negligible in the low energy regime $\rho \ll \sigma$; source terms, which come from the bulk perturbations and which cannot be determined solely from the evolution inside the brane.

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